**Summary**

Two-stage XFEL (seeding option) generating 0.86 Å x-rays (14.4 keV) with a spectral bandwidth of $\approx 20$ meV [1] is proposed for the x-ray laboratory integrated into the TESLA linear collider project. Special interest for the 14.4 keV x-rays is due to many additional possibilities which the powerful and diverse nuclear resonance scattering techniques with the highly monochromatic 14.4 Mossbauer radiation open for studies of structure and dynamics of solids, biological molecules, etc [2].

A design of the single-pass two-stage x-ray self-amplified spontaneous emission (SASE) FEL is used [3]. The generation of the narrow-band x-rays in an XFEL undulator is boosted by a radiation seed of the required spectral width which is produced in another XFEL undulator and monochromatized to the required level by an x-ray monochromator. Significant advantage of such a two-stage XFEL over the single-stage XFEL is not only the $\approx 10^3$ times narrower energy bandwidth at the same peak power and thus much higher spectral flux, brilliance, and nearly macroscopic $\approx 30 \mu \text{m}$ coherence length, but also significantly suppressed fluctuations of the spectral power density.

**Introduction**

The SASE FEL [4, 5] can be modified as proposed in [3] to reduce significantly the bandwidth and the fluctuations of the output radiation. The modified scheme consists of two undulators and an x-ray monochromator located between them - Fig. 1. The first undulator operates in the linear regime of amplification starting from noise and the output radiation has the usual SASE properties. After the first undulator the electron beam is guided through a bypass and the x-ray beam enters the monochromator which selects a narrow band of radiation. At the entrance of the second undulator the monochromatic x-ray beam is combined with the electron beam and is amplified up to the saturation level.

The electron micro-bunching induced in the first undulator should be destroyed prior to its arrival at the second one. This is achieved automatically due to the natural energy spread of the electron beam guided through the bypass. At the entrance of the second undulator the
radiation power from the monochromator dominates significantly over the shot noise and the residual electron bunching, and the input signal bandwidth is small with respect to the FEL amplifier bandwidth.

The two-stage scheme possesses two significant advantages. First, it opens a perspective to achieve monochromaticity of the output radiation close to the limit given by the finite duration of the electron pulse and thus to increase its brilliance and coherence length. Second, shot-to-shot fluctuations of the energy spectral density could be reduced from 100 % to less than 10 % when the second undulator section operates at saturation. Since it is a single bunch scheme, it does not require any special time diagram of the accelerator operation.

The conditions that are necessary and sufficient for the effective operation of a two-stage SASE FEL were discussed in [3] and can be summarized as follows

\[
P_{\text{in}}^{(2)} / P_{\text{shot}} = G^{(1)} R_m (\delta \lambda/\lambda)_m / (\delta \lambda/\lambda)_{\text{SASE}} \gg 1 ,
\]

\[
\lambda/\pi \sigma_z < (\delta \lambda/\lambda)_m < (\delta \lambda/\lambda)_{\text{SASE}} , 
\]

\[
G^{(1)} \ll G_{\text{sat}}(\text{SASE}) .
\]

Here \( P_{\text{in}}^{(2)} \) is the input radiation power at the entrance to the second undulator, \( P_{\text{shot}} \) is the effective power of shot noise, \( G^{(1)} \) is the power gain in the first undulator, \( R_m \) is the integral reflection coefficient of the mirrors and the dispersive elements of the monochromator, \( (\delta \lambda/\lambda)_m \) is the resolution of the monochromator, \( (\delta \lambda/\lambda)_{\text{SASE}} \) is the radiation bandwidth of the SASE FEL at the exit of the first undulator, \( \sigma_z \) is the rms length of the electron bunch, and \( G_{\text{sat}}(\text{SASE}) \) is the power gain of SASE FEL at saturation.

An application of such a two-stage scheme to the 6 nm SASE FEL at the TESLA Test Facility at DESY [6] was discussed in [3, 7]. Now it is funded and is expected to be the main option for operation at the user facility.

Here we propose a design of the two-stage FEL generating 0.86 Å x-rays (14.4 keV) for the x-ray laboratory integrated into the TESLA linear collider project [8]. The undulators of the FEL under discussion have a fixed gap optimized for generation of the 14.4 keV x-rays. X-ray energy will be possible to tune in the 2 to 4 keV range by changing the energy of the electron beam.
Table 1: Parameters of the electron beam and the undulators

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, $\varepsilon_0$</td>
<td>25 GeV</td>
</tr>
<tr>
<td>Peak current, $I_0$</td>
<td>5 kA</td>
</tr>
<tr>
<td>rms bunch length, $\sigma_z$</td>
<td>23 $\mu$m</td>
</tr>
<tr>
<td>Normalized rms emittance, $\epsilon_n$</td>
<td>1.6$\pi$ mm mrad</td>
</tr>
<tr>
<td>rms energy spread (entrance)</td>
<td>2.5 MeV</td>
</tr>
<tr>
<td>External $\beta$-function</td>
<td>45 m</td>
</tr>
<tr>
<td>Bunch separation</td>
<td>93 ns</td>
</tr>
<tr>
<td>Number of bunches per train</td>
<td>11315</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>5 Hz</td>
</tr>
<tr>
<td><strong>Undulator</strong></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Planar</td>
</tr>
<tr>
<td>Period, $\lambda_w$</td>
<td>4.5 cm</td>
</tr>
<tr>
<td>Peak magnetic field, $H_w$</td>
<td>9.5 kGs</td>
</tr>
</tbody>
</table>

Special interest for the 14.4 keV x-rays is due to additional possibilities which the powerful and diverse nuclear resonance scattering techniques with the highly monochromatic 14.4 Mössbauer radiation open for studies of structure and dynamics of solids, biological molecules, etc [2].

**Parameters of the two-stage x-ray FEL**

Main parameters of the electron beam and the undulators are presented in Table 1 and coincide with those of the usual (single-stage) SASE FEL at 14.4 keV being designed for the TESLA facility. The SASE FEL bandwidth at the exit of the first stage is about $7 \times 10^{-4}$ and weakly depends on the gain. We require the monochromator FWHM bandwidth to be about 20 meV, or $1.4 \times 10^{-6}$ (see Eq. (2)). The integral reflection coefficient of all the crystals of the monochromator is expected to be in a range of 0.3 - 0.5. Requiring the excess of the input radiation power at the entrance to the second undulator $P_{\text{in}}^{(2)}$ over the effective power of the shot noise $P_{\text{shot}}$ to be two orders of magnitude (see (1) and [7]) we end up with a required gain of $1.5 \times 10^6$ in the first undulator. The SASE FEL gain at the saturation would be about $4 \times 10^6$ so that the condition (3) is satisfied.

Parameters of the first and the second stages are presented in Table 2. They have been calculated with the FEL simulation code FAST [9]. The growth of energy spread in the electron beam due to the quantum fluctuations of undulator radiation was taken into account [10, 11]. The peak and average brilliance of the x-ray beam at the exit of the second stage are 500 times larger than in the case of the single-stage SASE FEL. The shot-to-shot fluctuations of the energy spectral density are reduced to the 10% level due to nonlinear stabilization mechanism [7].

The distance between the two undulators is mainly defined by parameters of the electron beam bypass (chicane) that must compensate a path delay of x-rays in the monochromator.
Table 2: Parameters of the first and the second stages

<table>
<thead>
<tr>
<th>Stage</th>
<th>Wavelength, $\lambda$</th>
<th>Effective power of shot noise, $P_{\text{shot}}$</th>
<th>Length of undulator, $L_w^{(1)}$</th>
<th>FWHM bandwidth, $(\delta\lambda/\lambda)_{\text{SASE}}$</th>
<th>Radiation spot size (FWHM)</th>
<th>Angular divergence (FWHM)</th>
<th>Peak power</th>
<th>Average power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.860 Å</td>
<td>5 kW</td>
<td>140 m</td>
<td>$7 \times 10^{-1}$</td>
<td>50 $\mu$m</td>
<td>1 $\mu$rad</td>
<td>0.75 GW</td>
<td>4 W</td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5 MW</td>
<td>110 W</td>
</tr>
</tbody>
</table>

The latter is of an order of 1 cm (c.f. Sec. ). For a bending angle of the chicane magnets of 1.3$\times$ the total length of the chicane is about 40 m. The electron beam micro-bunching is completely destroyed at the end of the bypass due to the uncorrelated energy spread in the beam and reasonable longitudinal dispersion of the chicane [3].

**High energy-resolution, high heat-load, tunable x-ray monochromator**

The main requirements to the x-ray monochromator of the two stage XFEL are:

i. degree of monochromatization: $\lambda/\delta\lambda = E/\delta E = 0.7 \times 10^6$;

ii. tunability range: a few keV;

iii. resistance to the high heat-load $\approx 500$ W/mm$^2$.

To reach the required value of monochromatization alone is not a problem. Nowadays a monochromatization of $10^7$ and more is possible. Bragg diffraction is the main tool used for such purposes. For a recent review of the techniques used and achievements in this field see, e.g., [12]. However, the combination of the three requirements renders the realization of such a monochromator not so straightforward.
Table 3: Parameters of the monochromator and the electron beam bypass

<table>
<thead>
<tr>
<th><strong>Monochromator</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal energy</td>
<td>14.4 keV</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>20 meV</td>
</tr>
<tr>
<td>Tunability range</td>
<td>2-4 keV</td>
</tr>
<tr>
<td>Total reflection coefficient</td>
<td>0.3 - 0.5</td>
</tr>
<tr>
<td>Absorbed average power</td>
<td>&lt; 200 mW</td>
</tr>
<tr>
<td>Absorbed average power density</td>
<td>&lt; 50 W/mm²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Electron beam bypass (chicane)</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total length</td>
<td>40 m</td>
</tr>
<tr>
<td>Bending angle of magnets</td>
<td>1.3°</td>
</tr>
<tr>
<td>Path lengthening</td>
<td>1 cm</td>
</tr>
</tbody>
</table>

**Spectral width of Bragg reflections and tunability range.**

Tunability of an x-ray monochromator for a given monochromaticity will be addressed first. The relative energy width of a Bragg reflection in a thick nonabsorbing crystal (like silicon, diamond, etc) is given in the dynamical theory of diffraction in perfect crystals (see, e.g., [13]) by

\[ \frac{\delta E}{E} = \frac{\delta \lambda}{\lambda} = \frac{|\chi_{g}|}{\sin^2 \theta}. \]

Here \( \theta \) is the glancing angle of the radiation plane wave to the reflecting atomic planes \((hkl)\) with the interplanar distance \(d_{hkl}\) and the related reciprocal vector \(g\) where \(|g| = 2\pi/d_{hkl}\).

The relation between the wavelength \(\lambda\) of the reflected x rays and \(\theta\) is given by the Bragg law \(2d_{hkl} \sin \theta = \lambda\).

\[ \chi_{g} = -\frac{r_e \lambda^2}{\pi V} Z f\left(\frac{\sin \theta}{\lambda}\right) \exp\left(-\frac{\langle u^2 \rangle}{\lambda^2} 8\pi^2 \sin^2 \theta\right) \]

is the Fourier component of the electric susceptibility corresponding to the reciprocal vector \(g\). The expression is valid for a single atom crystal. The following notations are used: \(V\) is the volume of the crystal unit cell; \(Z\) is atomic number; \(r_e\) is the classical electron radius; \(f(\ldots)\) is the atomic scattering form-factor; and \(\exp(\ldots)\) is the Debye-Waller factor with \(\langle u^2 \rangle\) as the mean square displacement of atoms in the direction of the scattering vector \(g\) due to thermal vibrations.

The combination of the both equations gives

\[ \frac{\delta E}{E} = \frac{r_e \lambda_{hkl}^2}{\pi V} Z f\left(\lambda_{hkl}^{-1}\right) \exp\left(-8\pi^2 \frac{\langle u^2 \rangle}{\lambda_{hkl}^2}\right). \] (4)

Here the Bragg wavelength \(\lambda_{hkl} = 2d_{hkl}\) is introduced - the largest wavelength of x-rays allowed to be reflected from the \((hkl)\) atomic planes by the Bragg law.

An important and very favourable implication of eq. (4) for our applications is that the relative spectral width for the given Bragg reflection \((hkl)\) is independent of the energy or glancing angle of x-rays and defined merely by properties of the crystal and reflecting atomic
planes. In particular it implies that the choice of a crystal, reflecting atomic planes and crystal temperature determines the spectral resolution. Figure 2 shows results of evaluations of the monochromaticity $E / \delta E$ of x-rays reflected from different atomic planes $(hkl)$ in diamond (C) and silicon (Si) single crystals at room temperature. The range of tunability is limited only by the lowest x-ray energy allowed by the Bragg law to be reflected from the $(hkl)$ atomic planes.

The $1 \mu$rad divergency (FWHM) of x-rays from the first undulator were also taken into account, which shows up in the decreasing monochromaticity with raising x-ray energy. This occurs when the angular acceptance of Bragg reflections approaches the angular divergence of the incoming beam.

As it is seen from Fig. 2 the number of possible reflections which provide required monochromaticity and tunability range is rather limited. In case of diamond (C), these are $(137)$ or $(117)$ and equivalent ones. In case of silicon single crystals these are $(139)$ or $(339)$ and equivalent ones.

We are discussing here only silicon and diamond single crystals. There are two reasons for this. Si single crystals are the most perfect crystals available nowadays. This is an important feature which ensures the preservation of the coherent properties of the radiation from the first undulator. Diamond although not so perfect as silicon, nevertheless sufficiently large $\approx 10 \times 10 \times 1 \text{ mm}^3$ perfect crystals are available already now [14, 15]. The greatest
advantage of diamond is its ability to withstand the high heat load due to the extremely high thermal conductivity, low thermal expansion, small x-ray absorption, and high reflectivity.

**Actual schemes**

We have chosen the 4-bounce scheme of the x-ray monochromator as shown in Fig. 4. This solution is advantageous as it allows to keep the direction and the position of the x-ray beam at the exit the same as at the entrance of the monochromator.

![Figure 3: The two-stage XFEL with the 4-bounce x-ray monochromator: C(004) × C(004) – C(137) × C(137). Additional path lag acquired by the x-rays in the monochromator is δL = 3.0H.](image)

This solution is advantageous also due to the possibility to use the first two Bragg reflectors as a high-heat load premonochromator, which withdraws the major heat load from the actual high energy-resolution monochromator - the third and the forth crystals. In the pre-monochromator part one can use, e.g., diamond crystal plates of a 100 μm thickness and the reflection C(004). Given the crystal is perfect, it reflects 99% of the incident x-rays within a band of 132 meV. Only 5% of the off-band radiation is absorbed, and the rest passes through. The absorbed power is thus 20 times less than the incident one and is about 200 mW. The absorbed power density is about 50 W/mm². It is comparable with that at the monochromators of the 3rd generation synchrotron sources [16, 17, 18]. The radiation power which reaches
the high resolution monochromator crystals is \( \approx 1.3\% \) of the initial value. The latter can be reduced by a factor of two if to use the reflection C(133) with a bandwidth of 75 meV.

The final monochromatization to the required level takes place by a high-index reflection in the third and the forth crystals. The required monochromatization \( E/\delta E = 0.7 \times 10^6 \) of the 14.4 keV x-rays can be achieved, according to Fig. 2, by only a very limited number of reflections. The final choice of the reflection should be dictated by the requirements of tunability and heat-load. The Bragg reflections in diamond has a smaller tunability range. On the other hand they have higher reflectivity and angular acceptance. Fine adjustment of the angular acceptance and the energy bandwidth can be performed by using asymmetric Bragg reflections.

An important technical issue is a path delay (with respect to the straight path) which the x-ray pulse acquires in the monochromator. The path delay equals to

\[
\delta L = H \left( \tan \theta_{HKL} + \tan \theta_{hkl} \right),
\]

where \( H \) is the beam shift, \( \theta_{HKL} \) in the Bragg angle of the reflections in the high heat-load part (the first two crystals), and \( \theta_{hkl} \) in the Bragg angle of the reflections in the high energy-resolution part of the monochromator (the third and the forth crystals). By varying \( H \) one can keep the delay \( \delta L \) constant in the whole tunability range of the monochromator. For the proposed monochromator schemes the actual values of \( \delta L \) are given in Fig. 4 caption and can be about 1 cm.

**Conclusion**

Our analysis shows that the construction of the high-brilliant two-stage SASE FEL in the Angström spectral range is feasible. We have considered an extreme case when the spectral bandwidth is defined by the length of the electron bunch (lower limit in (2)). By increasing the bandwidth one can increase the tunability range and reduce the power density incident on crystals of the monochromator. The final choice of parameters will be dictated by needs of potential users of intense monochromatic x-rays.

**References**


