Higher Order Modes

The longitudinal field

Monopole

Dipole

Quadrupole
Dipole Mode

\( E_z = J_1(k_z r) \cos \phi e^{-i k_{zz} z} \)

\( E_r = -\frac{k_z}{2k_r} \left[ J_0(k_z r) - i J_2(k_z r) \right] \cos \phi e^{-i k_{zz} z} \)

Close to the center this can be approximated by

\( E_z \approx \frac{k_z}{2} \frac{r}{r} \cos \phi e^{-i k_{zz} z} \)

\( E_r \approx -\frac{k_z}{4} \cos \phi e^{-i k_{zz} z} \)
Dipole Mode

A loss factor can be defined

\[ k_d = \left( \frac{\omega}{c} \right)^2 \frac{\omega}{4} \left( \frac{R}{Q} \right) \]  \hspace{1cm} (3)

With the energy lost by a bunch

\[ U_q = k_d q^2 \rho^2 \quad \rho = \text{offset} \]  \hspace{1cm} (4)

and the transverse kick

\[ V_\perp = \frac{\omega^2}{2c} \left( \frac{R}{Q} \right) \rho q. \]  \hspace{1cm} (5)

⇒ Study modes simultaneously with HOM-coupler and BPM.
Dipole Mode

Definition of $Q_L$ from Energy

$$Q_L = \frac{\omega U}{P_d} \quad (6)$$

(Energy) time constant

$$\tau_E = \frac{\omega}{Q_L} \quad (7)$$

Then the power coupled out of the cavity is

$$P_d = \frac{U}{\tau_E} = k_d q^2 \rho^2 f_b \quad (8)$$
Observe beam deflection after modulation has been turned off

Beam is on

Beam is modulated

We measure this amplitude

f_mod = 21.09959 MHz

→ a HOM is excited

Q = 53000

f_mod = 21.14959 MHz

No mode is excited
Beam current measured from beam loading and in BPM 2BC2

maximum at 14.5 mV
Locate the HOM
A second trigger for $\tau$ is required.
2 kHz OFF
beam current 2 mA, modulation depth 20 %, 1 V ≈ 2.2 mm

HOM Scan 9-Mar-2001

Step size 1 kHz

about 20 HOM candidates with strong deflection
(dipole D mode and quadrupole mode Q assignment - preliminary)
about 60 other excited modes with weak deflection
Examples of time domain spectra:

- The excited mode is present in all cavities.
- Several modes in different cavities are excited.
Modulation Frequency: 24.205 MHz

![Graph showing HOM power (dBm) over time (s) with various cavity coupled frequencies.](image-url)